GATE PATHSHALA

Strength Of Materials
(Assignment-01: Solutions)

Author Name: Krishna Teja

April 9, 2025

Question - 01

A rod 250 cm long and of diameter 4.0 cm is subjected to an axial pull of 35 kN. If the Young's modulus of the material of the rod is 2.5×10^5 N/mm^2, determine:

1. Stress
2. Strain
3. Elongation of the rod

Solution- 01

Given Data

$$L = 250 \text{ cm} = 2500 \text{ mm},$$
$$d = 4.0 \text{ cm} = 40 \text{ mm},$$
$$P = 35 \text{ kN} = 35000 \text{ N},$$
$$E = 2.5 \times 10^5 \text{ N/mm}^2.$$

Calculate the Stress

The axial stress is given by:

$$\sigma = \frac{P}{A},$$

where A is the cross-sectional area of the rod:

$$A = \frac{\pi d^2}{4} = \frac{\pi (40)^2}{4}$$
$$= \frac{\pi \times 1600}{4} = \frac{5026.55}{4} = 1256.64 \text{ mm}^2.$$

Now, calculating the stress:

$$\sigma = \frac{35000}{1256.64} = 27.84 \text{ N/mm}^2.$$

Calculate the Strain

Strain is given by:

$$\epsilon = \frac{\sigma}{E}.$$

Substituting values:

$$\epsilon = \frac{27.84}{2.5 \times 10^5} = \frac{27.84}{250000} = 0.0001114.$$

Calculate the Elongation of the Rod

The elongation is given by:

$$\Delta L = \epsilon \times L.$$

Substituting values:

$$\Delta L = 0.0001114 \times 2500 = 0.2785 \text{ mm} = 0.02785 \text{ cm}.$$

Question - 02

Find the Young's modulus of a rod of diameter 30 mm and of length 300 mm which is subjected to a tensile load of 60 kN and the extension of the rod is equal to 0.4 mm.

Solution- 02

Given Data

$$\begin{aligned} d &= 30 \text{ mm}, \\ L &= 300 \text{ mm}, \\ P &= 60 \text{ kN} = 60000 \text{ N}, \\ \Delta L &= 0.4 \text{ mm}. \end{aligned}$$

Calculate the Stress

Axial stress is given by:

$$\sigma = \frac{P}{A},$$

where A is the cross-sectional area of the rod:

$$\begin{aligned} A &= \frac{\pi d^2}{4} = \frac{\pi (30)^2}{4} \\ &= \frac{\pi \times 900}{4} = \frac{2827.43}{4} = 706.86 \text{ mm}^2. \end{aligned}$$

Now, calculating the stress:

$$\sigma = \frac{60000}{706.86} = 84.89 \text{ N/mm}^2.$$

Calculate the Strain

Strain is given by:

$$\epsilon = \frac{\Delta L}{L}.$$

Substituting values:

$$\epsilon = \frac{0.4}{300} = 0.00133.$$

Calculate Young's Modulus

Young's modulus is given by:

$$E = \frac{\sigma}{\epsilon}.$$

Substituting values:

$$E = \frac{84.89}{0.00133} = 63.6 \times 10^3 \text{ N/mm}^2.$$

$$E = 63.6 \text{ GN/m}^2.$$

Final Answer

$$\mathbf{E = 63.6 \text{ GN/m}^2}$$

Question - 03

A rectangular element in a linearly elastic isotropic material is subjected to tensile stresses of 83 N/mm^2 and 65 N/mm^2 on mutually perpendicular planes. Determine the strain in the direction of each stress and in the direction perpendicular to both stresses.

$$Take, E = 200000 \,\text{N/mm}^2, \quad v = 0.3$$

Solution- 03

Given Data

$$\begin{aligned}
\sigma_x &= 83 \text{ N/mm}^2, \\
\sigma_y &= 65 \text{ N/mm}^2, \\
\tau_{xy} &= 0, \\
E &= 200000 \text{ N/mm}^2, \\
v &= 0.3.
\end{aligned}$$

Normal Strains in the x and y Directions

Using the strain formula:

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - v\sigma_y\right),$$
$$\epsilon_y = \frac{1}{E}\left(\sigma_y - v\sigma_x\right).$$

Substituting values:

$$\epsilon_x = \frac{1}{200000}\left(83 - (0.3 \times 65)\right) = 3.175 \times 10^{-4},$$
$$\epsilon_y = \frac{1}{200000}\left(65 - (0.3 \times 83)\right) = 2.005 \times 10^{-4}.$$

Strain in the Perpendicular Direction (z-direction)

$$\epsilon_z = -\frac{v}{E}(\sigma_x + \sigma_y)$$
$$= -\frac{0.3}{200000}(148) = -2.22 \times 10^{-4}.$$

Question - 04

From the information given in Question-03. Determine the Principal Strains and Maximum Shear Strain

Solution- 04

Principal Strains

The principal strains are given by:

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}.$$

Since $\gamma_{xy} = 0$,

$$\epsilon_{1,2} = \frac{3.175 \times 10^{-4} + 2.005 \times 10^{-4}}{2} \pm \sqrt{\left(\frac{3.175 \times 10^{-4} - 2.005 \times 10^{-4}}{2}\right)^2}.$$

$$\epsilon_1 = 3.175 \times 10^{-4},$$
$$\epsilon_2 = 2.005 \times 10^{-4}.$$

Maximum Shear Strain

$$\gamma_{\max} = \epsilon_1 - \epsilon_2 = 1.17 \times 10^{-4}.$$

Question - 05

A structural member supports loads which produce, at a particular point, a direct tensile stress of 80 N/mm^2 and a shear stress of 45 N/mm^2 on the same plane. Calculate the values of the principal stresses at the point and also the maximum shear stress, stating on which planes this will act.

Solution- 05

Given Data

$$\begin{aligned} \sigma_x &= 80 \text{ N/mm}^2 \\ \sigma_y &= 0 \text{ (No normal stress in the perpendicular direction)} \\ \tau_{xy} &= 45 \text{ N/mm}^2 \end{aligned}$$

Principal Stresses

The principal stresses are given by:

$$\sigma_{I,II} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substituting the given values:

$$\begin{aligned} \sigma_{I,II} &= \frac{80+0}{2} \pm \sqrt{\left(\frac{80-0}{2}\right)^2 + 45^2} \\ &= 40 \pm \sqrt{40^2 + 45^2} \\ &= 40 \pm \sqrt{1600 + 2025} \\ &= 40 \pm \sqrt{3625} \\ &= 40 \pm 60.2 \end{aligned}$$

Thus, the principal stresses are:

$$\begin{aligned} \sigma_I &= 100.2 \text{ N/mm}^2 \\ \sigma_{II} &= -20.2 \text{ N/mm}^2 \end{aligned}$$

Maximum Shear Stress

The maximum shear stress is given by:

$$\tau_{\max} = \frac{\sigma_I - \sigma_{II}}{2}$$

Substituting the values:

$$\tau_{\max} = \frac{100.2 - (-20.2)}{2} = \frac{120.4}{2} = 60.2 \text{ N/mm}^2$$

The maximum shear stress acts at 45° to the principal planes.

Final Answers

- **Principal Stresses:**

$$\sigma_I = 100.2 \text{ N/mm}^2$$

$$\sigma_{II} = -20.2 \text{ N/mm}^2$$

- **Maximum Shear Stress:** $\tau_{\max} = 60.2 \text{ N/mm}^2$ at $45°$ to principal planes.

Question - 06

A circular rod 0.2 m long tapers from 20 mm diameter at one end to 10 mm diameter at the other. On applying an axial pull of 6 kN, it was found to extend by 0.068 mm. Find the Young's modulus of the material of the rod.

Solution- 06

Given:

$$L = 0.2m$$
$$d_1 = 0.02m$$
$$d_2 = 0.01m$$
$$P = 6000N$$
$$\Delta L = 0.000068m$$

Formula for elongation of a tapered circular rod:

$$\Delta L = \frac{4PL}{\pi E d_1 d_2}$$

Substitute the given values:

$$0.000068 = \frac{4 \times 6000 \times 0.2}{\pi \times E \times 0.02 \times 0.01}$$

Simplify numerator:

$$4 \times 6000 \times 0.2 = 4800$$

Simplify denominator (excluding E):

$$\pi \times 0.02 \times 0.01 = 6.2832 \times 10^{-4}$$

Rewriting the equation:

$$0.000068 = \frac{4800}{6.2832 \times 10^{-4} \times E}$$

Solving for E:

$$E = \frac{4800}{6.2832 \times 10^{-4} \times 0.000068}$$

$$E = \frac{4800}{4.272576 \times 10^{-8}} \approx 112.3 \times 10^9 \text{ N/m}^2$$