GATE PATHSHALA
Properties and Derivatives of Functions

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Properties of Derivatives

Let $f(x)$, $g(x)$, and c be functions and constants.

1. Linearity

$$\frac{d}{dx}[cf(x)] = c \cdot f'(x)$$
$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

2. Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$u\,dv + v\,du = d(uv)$$

3. Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$
$$\frac{v\,du - u\,dv}{v^2} = d\left(\frac{u}{v}\right)$$

4. Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

If $y = f(u)$, and $u = g(x)$, then:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

5. Total Differential for Two Variables:

If $z = f(x, y)$, then:

$$dz = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy$$

6. Derivative of a Square:

$$d(x^2) = 2x\,dx \quad \text{or more generally} \quad d(u^2) = 2u\,du$$

7. Derivative of a Logarithm:

$$d(\ln x) = \frac{1}{x}\,dx$$

8. Derivative of an Exponential:

$$d(e^x) = e^x\,dx$$

9. Derivative of a Power Function:

$$d(x^n) = n\,x^{n-1}\,dx$$

10. Differential of a Function Composition:

$$d(f(g(x))) = f'(g(x))\,g'(x)\,dx$$

11. Product of Three Functions:

$$d(uvw) = uv\,dw + uw\,dv + vw\,du$$

List of Derivatives

A. Basic Functions

$$\frac{d}{dx}[c] = 0$$
$$\frac{d}{dx}[x] = 1$$
$$\frac{d}{dx}[x^n] = nx^{n-1}$$
$$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$
$$\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}$$

B. Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x$$
$$\frac{d}{dx}[\cos x] = -\sin x$$
$$\frac{d}{dx}[\tan x] = \sec^2 x$$
$$\frac{d}{dx}[\cot x] = -\csc^2 x$$
$$\frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

C. Inverse Trigonometric Functions

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$$
$$\frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}[\csc^{-1} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

D. Exponential and Logarithmic Functions

$$\frac{d}{dx}[e^x] = e^x$$
$$\frac{d}{dx}[a^x] = a^x \ln a$$
$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$
$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$$

E. Hyperbolic Functions

$$\frac{d}{dx}[\sinh x] = \cosh x$$
$$\frac{d}{dx}[\cosh x] = \sinh x$$
$$\frac{d}{dx}[\tanh x] = \text{sech}^2 x$$
$$\frac{d}{dx}[\coth x] = -\text{csch}^2 x$$
$$\frac{d}{dx}[\text{sech}\, x] = -\text{sech}\, x \tanh x$$
$$\frac{d}{dx}[\text{csch}\, x] = -\text{csch}\, x \coth x$$