GATE PATHSHALA

Fluid Flow Kinematics Formula Sheet

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April 8, 2025

1. Types of Fluid Motion

$$\text{Steady Flow:} \quad \frac{\partial}{\partial t}(\cdot) = 0$$
$$\text{Unsteady Flow:} \quad \frac{\partial}{\partial t}(\cdot) \neq 0$$
$$\text{Uniform Flow:} \quad \frac{\partial}{\partial x}(\cdot) = 0$$
$$\text{Non-uniform Flow:} \quad \frac{\partial}{\partial x}(\cdot) \neq 0$$

2. Velocity Field

$$\vec{V}(x, y, z, t) = u\,\hat{i} + v\,\hat{j} + w\,\hat{k}$$

3. Acceleration in a Fluid

Material derivative (total derivative):

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Acceleration components:

$$a_x = \frac{Du}{Dt}, \quad a_y = \frac{Dv}{Dt}, \quad a_z = \frac{Dw}{Dt}$$

4. Streamlines, Pathlines, Streaklines

Streamline:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Pathline: Actual path followed by a particle over time.
Streakline: Locus of all particles that passed through a fixed point.

5. Continuity Equation (Incompressible Flow)

2D:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

3D:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

6. Rotation (Vorticity)

Rotation (angular velocity):

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Vorticity vector:

$$\vec{\omega} = \nabla \times \vec{V}$$

7. Irrotational Flow Condition

$$\nabla \times \vec{V} = 0$$

8. Velocity Potential Function ϕ

For irrotational flow:

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}$$

Laplace's Equation:

$$\nabla^2 \phi = 0$$

9. Stream Function ψ (2D Incompressible)

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

Automatically satisfies:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

10. Flow Net

Orthogonal intersection of streamlines and equipotential lines
Used to visualize 2D potential flows

11. Velocity Components in Polar Coordinates

Let V_r and V_θ be the velocity components in the radial and tangential directions, respectively.

$$V_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$
$$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

12. Cauchy-Riemann Equations in Polar Coordinates

For incompressible and irrotational flow:

$$\frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \qquad \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

13. Laplace's Equations in Polar Coordinates

Both ϕ and ψ satisfy Laplace's equation:

For $\phi(r, \theta)$:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$$

For $\psi(r, \theta)$:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0$$