The ratio of specific heats for Various gases:

The ratio of specific heats is given by a special symbol γ due to its frequency of appearance in compressible flow analysis, i.e.,

$$\gamma \equiv \frac{c_p}{c_v} = \frac{c_v + R}{c_v} = 1 + \frac{1}{c_v/R}$$

In terms of the ratio of specific heats γ and R, we express c_p and c_v as

$$c_p = \frac{\gamma}{\gamma - 1}R$$

$$c_v = \frac{1}{\gamma - 1}R$$

The ratio of specific heats is related to the degrees of freedom of the gas molecules, n, via

$$\gamma = \frac{n+2}{n}$$

The degrees of freedom of a molecule are represented by the sum of the energy states that a molecule possesses. For example, atoms or molecules possess kinetic energy in three spatial directions. If they rotate as well, they have kinetic energy associated with their rotation. In a molecule, the atoms may vibrate with respect to each other, which then creates kinetic energy of vibration as well as the potential energy of intermolecular forces. Finally, the electrons in an atom or molecule are described by their own energy levels (both kinetic energy and potential) that depend on their position around the nucleus. As the temperature of the gas increases, the successively higher energy states are excited; thus the degrees of freedom increases. A monatomic gas, which may be modelled as a sphere, has at least three degrees of freedom, which represent translational motion in three spatial directions. Hence, for a monatomic gas, under "normal" temperatures the ratio of specific heats is

$$\gamma = \frac{5}{3} \cong 1.667 \quad \text{Monatomic gas at "normal" temperatures}$$

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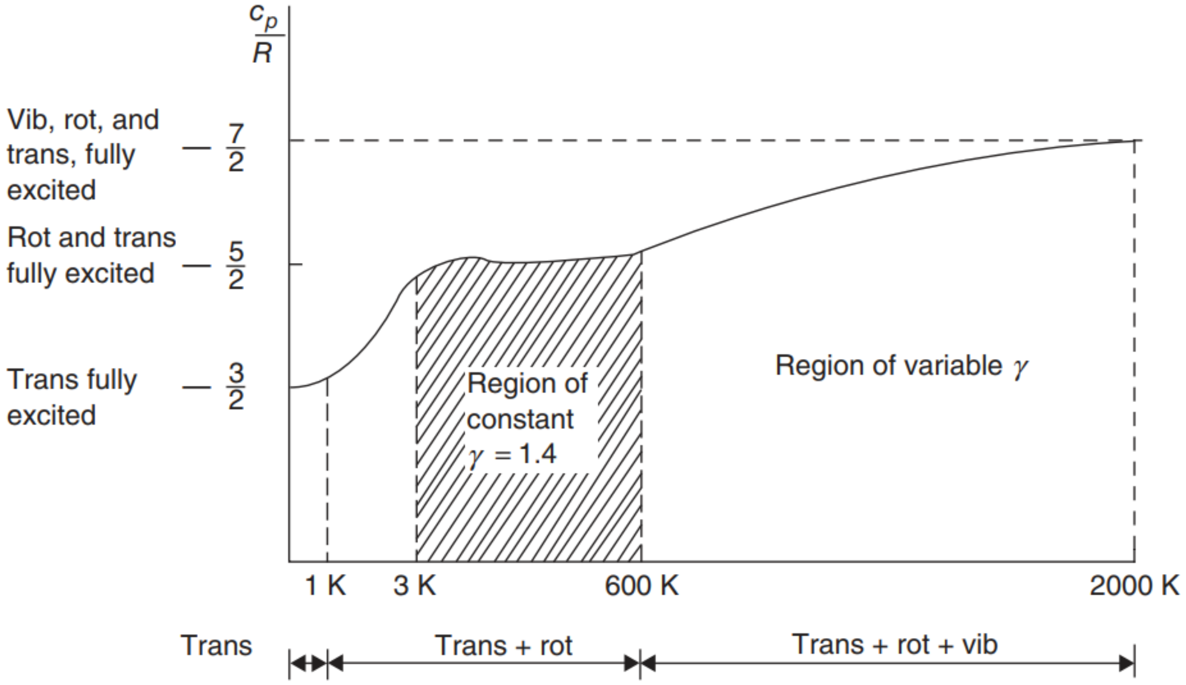
A monatomic gas has negligible rotational energy about the axes that pass through the atom due to its negligible moment of inertia. A monatomic gas will not experience a vibrational energy, as vibrational mode requires at least two atoms. At higher temperatures, the electronic energy state of the gas is affected, which eventually leads to ionization of the gas. For a diatomic gas, which may be modeled as a dumbbell, there are five degrees of freedom under "normal" temperature conditions, three of which are in translational motion and two of which are in rotational motion. The third rotational motion along the intermolecular axis of the dumbbell is negligibly small. Hence for a diatomic gas such as air (near room temperature), hydrogen, nitrogen, and so on, the ratio of specific heats is

$$\gamma = \frac{7}{5} = 1.4 \quad \text{Diatomic gas at "normal" temperatures}$$

At high temperatures, molecular vibrational modes and the excitation of electrons add to the degrees of freedom and that lowers γ. For example, at ~600 K vibrational modes in air are excited, thus the degrees of freedom of diatomic gasses are initially increased by 1, that is, it becomes 5 + 1 = 6, when the vibrational mode is excited. Therefore, the ratio of specific heats for diatomic gases at elevated temperatures becomes

$$\gamma = \frac{8}{6} \approx 1.33 \quad \text{Diatomic gas at elevated temperatures}$$



The vibrational mode represents two energy states corresponding to the kinetic energy of vibration and the potential energy associated with the intermolecular forces. When fully excited, the vibrational mode in a diatomic gas, such as air, adds 2 to the degrees of freedom, that is, it becomes 7. Therefore, the ratio of specific heats becomes

$$\gamma = \frac{9}{7} \approx 1.29 \quad \text{Diatomic gas at higher temperatures}$$

For example, air at 2000 K has its translational, rotational, and vibrational energy states fully excited. This temperature level describes the combustor or afterburner environment. Gases with a more complex structure than a diatomic gas have mote degrees of freedom, and thus their ratio of specific heats is less than 1.4. Figure 2.1 (from Anderson, 2003) shows the behavior of a diatomic gas from 0 to 2000 K. The nearly constant specific heat ratio between 3 and 600 K represents the calorically perfect gas behavior of a diatomic gas such as air with $\gamma = 1.4$. Note that near absolute zero (0 K), $c_v/R \rightarrow 3/2$; therefore, a diatomic gas ceases to rotate and thus behaves like a monatomic gas, that is, it exhibits the same degrees of freedom as a monatomic gas, that is, $n = 3$, $\gamma = 5/3$.